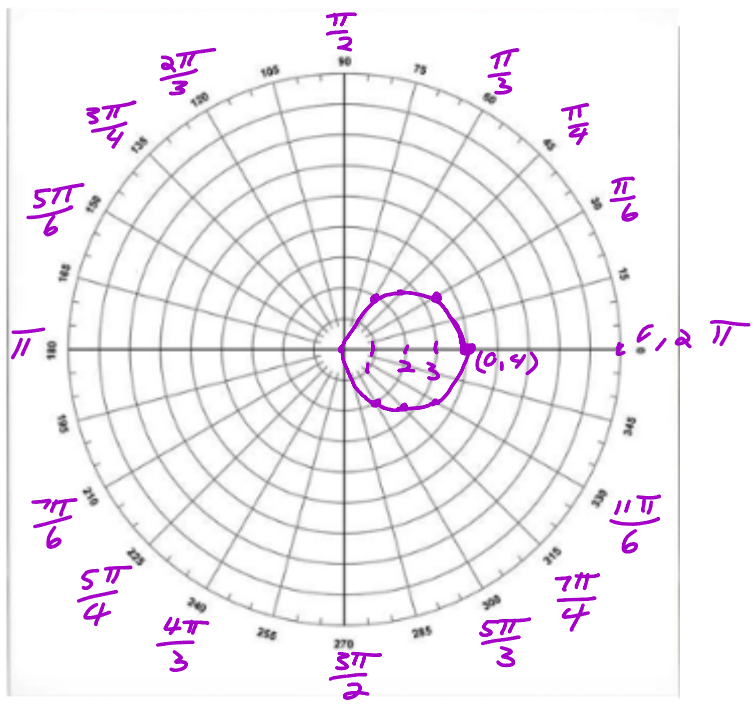
Period 4, Feb 20, 2025

Graph the polar equation $r = 4\cos\theta$ with θ in radians.

θ	r
0	$4 = 4\cdot\cos 0 = 4\cdot 1$
$\frac{\pi}{6}$	$2\sqrt{3} = 4\cos\frac{\pi}{6} = 4\cdot\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$2\sqrt{2} = 4\cos\frac{\pi}{4} = 4\cdot\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$2 = 4\cos\frac{\pi}{3} = 4\cdot\frac{1}{2}$
$\frac{\pi}{2}$	$0 = 4\cos\frac{\pi}{2} = 4\cdot 0$
$\frac{2\pi}{3}$	$-2 = 4\cos\frac{2\pi}{3} = 4\cdot -\frac{1}{2}$
$\frac{3\pi}{4}$	$-2\sqrt{2} = 4\cos\frac{3\pi}{4} = 4\cdot -\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$-2\sqrt{3} = 4\cos\frac{5\pi}{6} = 4\cdot -\frac{\sqrt{3}}{2}$
π	$-4 = 4\cos\pi = 4(-1)$
$\frac{7\pi}{6}$	$-2\sqrt{3} = 4\cos\frac{7\pi}{6} = 4\left(-\frac{\sqrt{3}}{2}\right)$
$\frac{5\pi}{4}$	$-2\sqrt{2} = 4\cos\frac{5\pi}{4} = 4\cdot -\frac{\sqrt{2}}{2}$
$\frac{4\pi}{3}$	$-2 = 4\cos\frac{4\pi}{3} = 4\left(-\frac{1}{2}\right)$
$\frac{3\pi}{2}$	$0 = 4\cos\frac{3\pi}{2} = 4(0)$
$\frac{5\pi}{3}$	$2 = 4\cos\frac{5\pi}{3} = 4\cdot\frac{1}{2}$
$\frac{7\pi}{4}$	$2\sqrt{2} = 4\cos\frac{7\pi}{4} = 4\cdot\frac{\sqrt{2}}{2}$
$\frac{11\pi}{6}$	$2\sqrt{3}$
2π	4

$2\sqrt{2} = 2.828$

$2\sqrt{3} = 3.464$

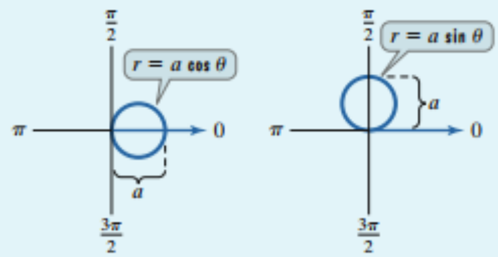


Circles in Polar Coordinates

The graphs of

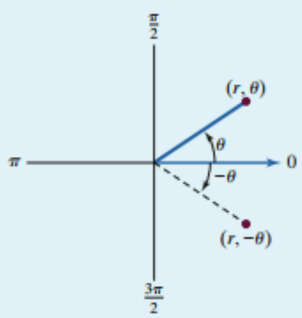
$$r = a\cos\theta \qquad \text{and} \qquad r = a\sin\theta,\ a > 0,$$

are circles.



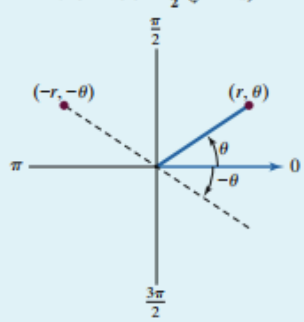
Tests for Symmetry in Polar Coordinates

Symmetry with Respect to the Polar Axis (x-Axis)



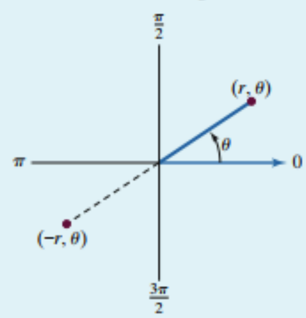
Replace θ with $-\theta$. If an equivalent equation results, the graph is symmetric with respect to the polar axis.

Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$ (y-Axis)



Replace (r, θ) with $(-r, -\theta)$. If an equivalent equation results, the graph is symmetric with respect to $\theta = \frac{\pi}{2}$.

Symmetry with Respect to the Pole (Origin)



Replace r with $-r$. If an equivalent equation results, the graph is symmetric with respect to the pole.

Check for symmetry and then graph the polar equation:

$$r = 1 - \cos\theta.$$

$r = 1 - \cos(-\theta)$

$r = 1 - \cos\theta$

$-r = 1 - \cos(-\theta)$

$-1 \cdot -r = (1 - \cos\theta) \cdot -1$

$r = -1 + \cos\theta$

$-1 \cdot -r = (1 - \cos\theta) \cdot -1$

$r = -1 + \cos\theta$

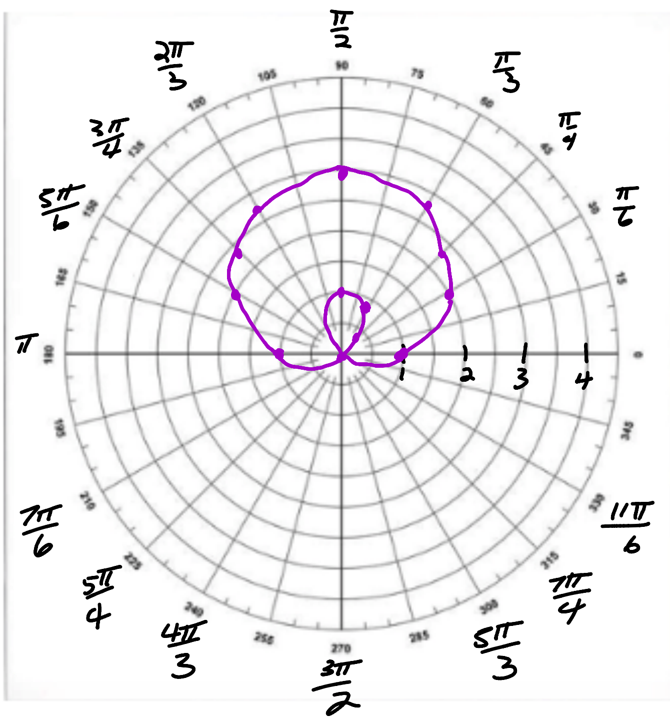
Graph the polar equation: $r = 1 + 2\sin\theta$.

$1+\sqrt{2} = 2.414$
$1+\sqrt{3} = 2.7$
$1-\sqrt{2} = -.414$
$1-\sqrt{3} = -.7$

Solution We first check for symmetry.

$$r = 1 + 2\sin\theta$$

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
Replace θ with $-\theta$.	Replace (r, θ) with $(-r, -\theta)$.	Replace r with $-r$.
$r = 1 + 2\sin(-\theta)$ $r = 1 + 2(-\sin\theta)$ $r = 1 - 2\sin\theta$	$-r = 1 + 2\sin(-\theta)$ $-r = 1 - 2\sin\theta$ $r = -1 + 2\sin\theta$	$-r = 1 + 2\sin\theta$ $r = -1 - 2\sin\theta$
No	nope	Nope



θ	r
0	$1 = 1+2\sin 0 = 1+2\cdot 0$
$\frac{\pi}{6}$	$2 = 1+2\sin\frac{\pi}{6} = 1+2\cdot\frac{1}{2}$
$\frac{\pi}{4}$	$1+\sqrt{2} = 1+2\sin\frac{\pi}{4} = 1+2\cdot\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$1+\sqrt{3} = 1+2\sin\frac{\pi}{3} = 1+2\cdot\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	$3 = 1+2\sin\frac{\pi}{2} = 1+2\cdot 1$
$\frac{2\pi}{3}$	$1+\sqrt{3} = 1+2\sin\frac{2\pi}{3} = 1+2\frac{\sqrt{3}}{2}$
$\frac{3\pi}{4}$	$1+\sqrt{2}$
$\frac{5\pi}{6}$	2
π	1
$\frac{7\pi}{6}$	$0 = 1+2\cdot\sin\frac{7\pi}{6} = 1+2(-\frac{1}{2})$
$\frac{5\pi}{4}$	$1-\sqrt{2} = 1+2\sin\frac{5\pi}{4} = 1+2\cdot-\frac{\sqrt{2}}{2}$
$\frac{4\pi}{3}$	$1-\sqrt{3} = 1+2\sin\frac{4\pi}{3} = 1+2(-\frac{\sqrt{3}}{2})$
$\frac{3\pi}{2}$	$-1 = 1+2\sin\frac{3\pi}{2} = 1+2(-1)$
$\frac{5\pi}{3}$	$1-\sqrt{3} = 1+2\sin\frac{5\pi}{3} = 1+2(-\frac{\sqrt{3}}{2})$
$\frac{7\pi}{4}$	$1-\sqrt{2}$
$\frac{11\pi}{6}$	0

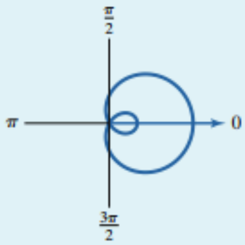
Limaçons

The graphs of

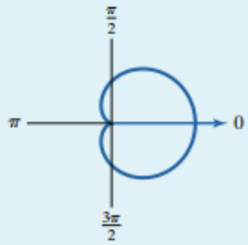
$$r = a + b\sin\theta, \quad r = a - b\sin\theta,$$
$$r = a + b\cos\theta, \quad r = a - b\cos\theta, \quad a > 0, b > 0$$

are called **limaçons**. The ratio $\frac{a}{b}$ determines a limaçon's shape.

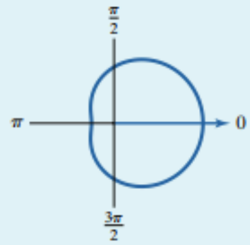
Inner loop if $\frac{a}{b} < 1$



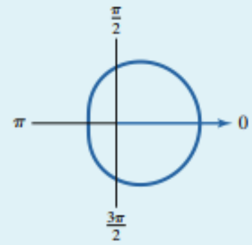
Heart-shaped (cardioids) if $\frac{a}{b} = 1$



Dimpled with no inner loop if $1 < \frac{a}{b} < 2$



No dimple and no inner loop if $\frac{a}{b} \geq 2$



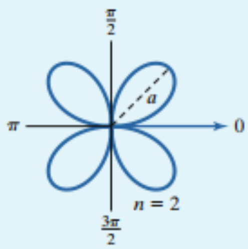
Rose Curves

The graphs of

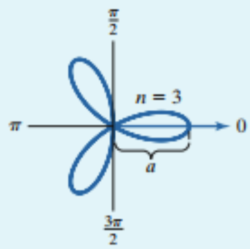
$$r = a\sin n\theta \quad \text{and} \quad r = a\cos n\theta, \quad a \neq 0,$$

are called **rose curves**. If n is even, the rose has $2n$ petals. If n is odd, the rose has n petals.

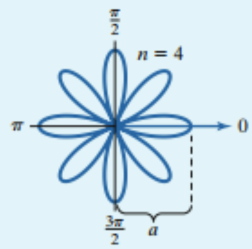
$r = a\sin 2\theta$
Rose curve with 4 petals



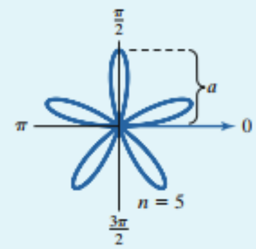
$r = a\cos 3\theta$
Rose curve with 3 petals



$r = a\cos 4\theta$
Rose curve with 8 petals



$r = a\sin 5\theta$
Rose curve with 5 petals



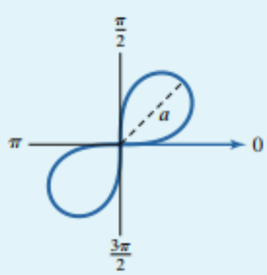
Lemniscates

The graphs of

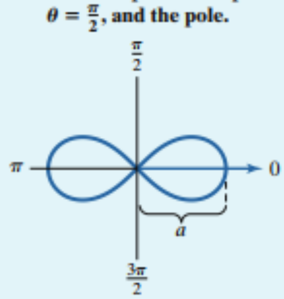
$$r^2 = a^2\sin 2\theta \quad \text{and} \quad r^2 = a^2\cos 2\theta, \quad a \neq 0$$

are called **lemniscates**.

$r^2 = a^2\sin 2\theta$
is symmetric with repect to the pole.



$r^2 = a^2\cos 2\theta$
is symmetric with repect to the polar axis, $\theta = \frac{\pi}{2}$, and the pole.



$\text{Cos}(-\theta) = \cos\theta$

$\text{Sin}(-\theta) = -\text{Sin}\,\theta$

$\csc(-\theta) = -\csc\theta$

$\text{Sec}(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

$\text{Tan}(-\theta) = -\text{Tan}\,\theta$

Graph the polar equation: $r = 4 \sin 2\theta$.

cut <'s in Half

Solution We first check for symmetry.

$$r = 4 \sin 2\theta$$

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
Replace θ with $-\theta$.	Replace (r, θ) with $(-r, -\theta)$.	Replace r with $-r$.
$r = 4 \sin 2(-\theta)$ $r = 4 \sin(-2\theta)$ $r = -4 \sin 2\theta$ None	$-r = 4 \sin 2(-\theta)$ $-r = 4 \sin(-2\theta)$ $-1 \cdot -r = -4 \sin 2\theta \cdot -1$ $r = 4 \sin 2\theta$ Passes	$-r = 4 \sin 2\theta$ $r = -4 \sin 2\theta$ None
Equation changes and fails this symmetry test.	Equation does not change.	Equation changes and fails this symmetry test.

θ	r
0	$0 = 4 \cdot \sin 0$
$\frac{\pi}{12}$	$4 \cdot \sin 2 \cdot \frac{\pi}{12}$ $2 = 4 \cdot \sin \frac{\pi}{6}$
$\frac{\pi}{8}$	$4 \cdot \sin 2 \cdot \frac{\pi}{8}$ $2\sqrt{2} = 4 \cdot \sin \frac{\pi}{4}$

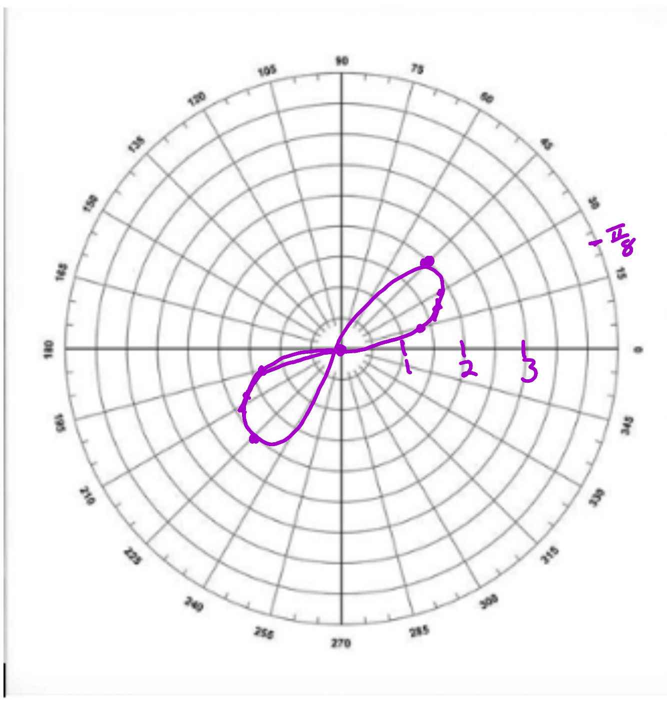
Graph the polar equation: $r^2 = 4 \sin 2\theta$.

Solution We first check for symmetry.

$$r^2 = 4 \sin 2\theta$$

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
Replace θ with $-\theta$.	Replace (r, θ) with $(-r, -\theta)$.	Replace r with $-r$.
$r^2 = 4 \sin 2(-\theta)$ $r^2 = 4 \sin(-2\theta)$ $r^2 = -4 \sin 2\theta$	$(-r)^2 = 4 \sin 2(-\theta)$ $r^2 = 4 \sin(-2\theta)$ $r^2 = -4 \sin 2\theta$	$(-r)^2 = 4 \sin 2\theta$ $r^2 = 4 \sin 2\theta$
Equation changes and fails this symmetry test.	Equation changes and fails this symmetry test.	Equation does not change.

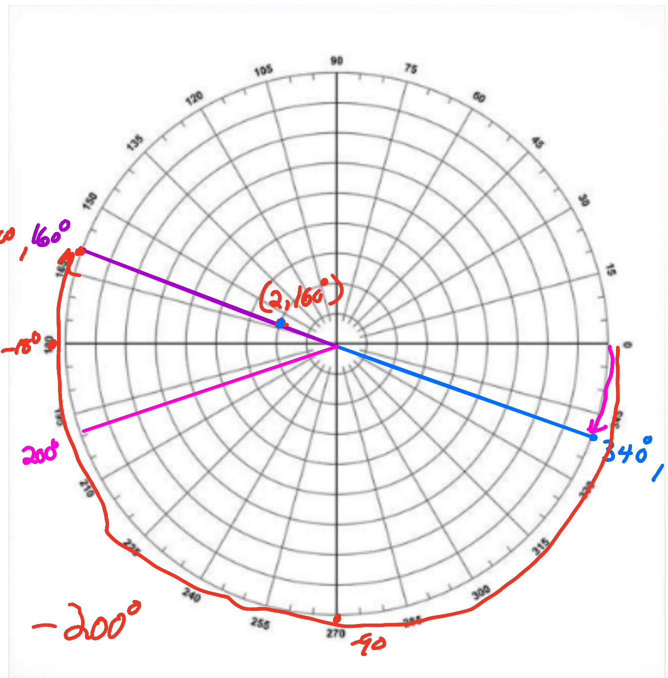
θ	$r^2 = 4 \sin 2\theta$
0	$0 = r^2 = 4 \cdot \sin 0$
$\frac{\pi}{12}$	$\pm\sqrt{2} = 4 \cdot \sin \frac{\pi}{6}$
$\frac{\pi}{8}$	$\pm\sqrt{2\sqrt{2}} \Leftarrow 4 \cdot \sin \frac{\pi}{4}$
$\frac{\pi}{6}$	$\pm\sqrt{2\sqrt{3}} \Leftarrow 4 \cdot \sin \frac{\pi}{6}$
$\frac{\pi}{4}$	$2 = 4 \cdot \sin \frac{\pi}{2}$



Select the representations that do not change the location of the point $(2,160^\circ)$.

Choose the correct answer below. Choose all that apply.

- [x] A. $(-2,340^\circ)$
- [ ] B. ~~$(-2,200^\circ)$~~
- [x] C. $(-2,-20^\circ)$
- [x] D. $(2,-200^\circ)$



Convert the polar equation to a rectangular equation. Then determine the graph's slope and y-intercept.

$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$r\sin\left(\theta+\frac{\pi}{4}\right)=9 \Rightarrow r\left[\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right]=9$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

The rectangular equation is $y=-x+9\sqrt{2}$.
(Type an equation. Type your answer in slope-intercept form. Simplify your answer, including any radicals. Use integers or fractions for any numbers in the equation. Rationalize all denominators. Do not factor.)

The graph's slope is -1.
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Rationalize the denominator.)

The graph's y-intercept is $9\sqrt{2}$.
(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression. Rationalize the denominator.)

$\frac{18\sqrt{2}}{2} = 9\sqrt{2}$

$r\left[\sin\theta\cdot\frac{\sqrt{2}}{2} + \cos\theta\cdot\frac{\sqrt{2}}{2}\right]=9$

$\frac{\sqrt{2}}{2}r\sin\theta + \frac{\sqrt{2}}{2}r\cos\theta = 9$

$\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}x = 9$

$-\frac{\sqrt{2}}{2}x \qquad -\frac{\sqrt{2}}{2}x$

$\frac{\cancel{\frac{\sqrt{2}}{2}}y}{\cancel{\frac{\sqrt{2}}{2}}} = \frac{-\frac{\sqrt{2}}{2}x+9}{\frac{\sqrt{2}}{2}}$

$y = \frac{-\cancel{\frac{\sqrt{2}}{2}}}{\cancel{\frac{\sqrt{2}}{2}}}\cdot x + \frac{\frac{9}{1}}{\frac{\sqrt{2}}{2}}$

$y = -1x + \frac{9}{1}\cdot\frac{2}{\sqrt{2}} = -x + \frac{18\sqrt{2}}{\sqrt{2}\sqrt{2}} = -x + 9\sqrt{2}$

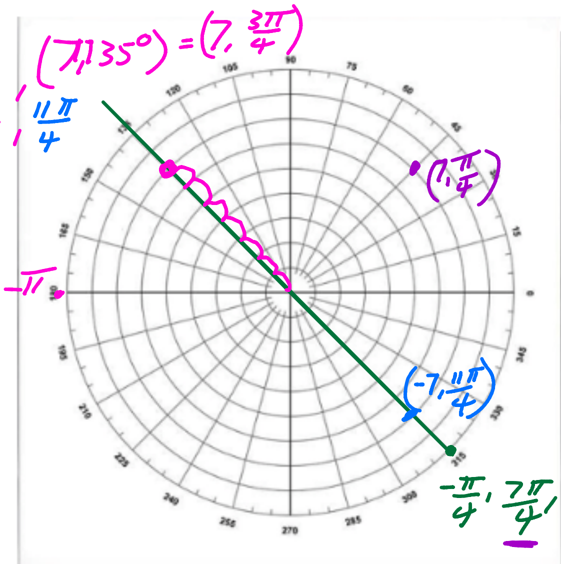
Select the representations that do not change the location of the point $\left(-7, -\frac{\pi}{4}\right)$.

$(-7, -45°)$

$\frac{11\pi}{4} = 2\frac{3}{4}\pi$

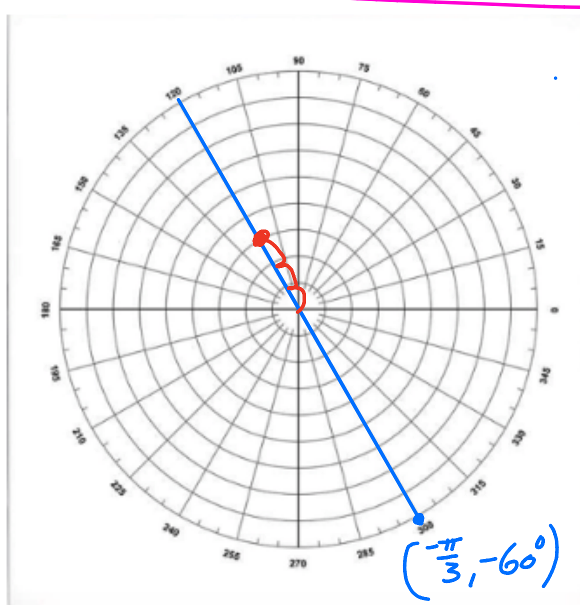
Choose the correct answer below. Choose all that apply.

- [x] A. $\left(-7, \frac{7\pi}{4}\right)$
- [ ] B. ~~$\left(7, \frac{\pi}{4}\right)$~~
- [ ] C. ~~$\left(-7, \frac{11\pi}{4}\right)$~~
- [x] D. $\left(7, -\frac{5\pi}{4}\right)$



Plot the point given in polar coordinates.

$\left(-3, -\frac{\pi}{3}\right)$



Convert the polar equation to a rectangular equation. Then use a rectangular coordinate system to graph the rectangular equation.

$r = 6\,\mathbf{cos}\,\theta + 2\,\mathbf{sin}\,\theta$

$r \cdot r = (6\cos\theta + 2\sin\theta)\cdot r$

$r^2 = 6r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = 6x + 2y$

$x^2 - 6x + 9 + y^2 - 2y + 1 = 0 + 9 + 1 \Rightarrow \underbrace{x^2 - 6x + 9} + y^2 - 2y + 1 = 10$

$a = 1$
$b = -6$
$\frac{b}{2} = \frac{-6}{2} = -3$
$\left(\frac{b}{2}\right)^2 = (-3)^2 = 9$

$a = 1$
$b = -2$
$\frac{b}{2} = \frac{-2}{2} = -1$
$\left(\frac{b}{2}\right)^2 = (-1)^2 = 1$

$\Rightarrow (x-3)^2 + (y-1)^2 = 10$

circle, center $(3,1)$ radius $= \sqrt{10}$

Convert the polar equation to a rectangular equation. Then use a rectangular coordinate system to graph the rectangular equation.

$r^2\,\mathbf{sin}\,2\theta = 76$

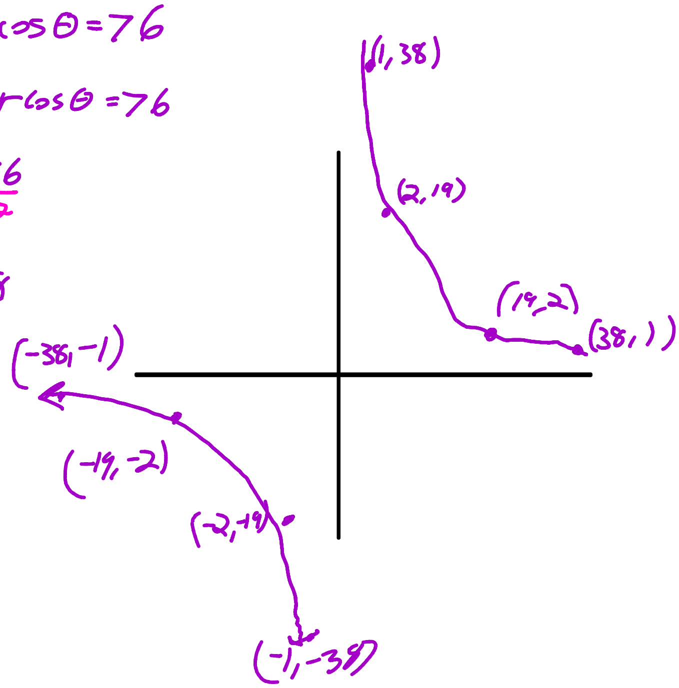
$r^2 \cdot 2\sin\theta\cos\theta = 76$

$2r\sin\theta \cdot r\cos\theta = 76$

$\frac{\cancel{2}y \cdot x}{\cancel{2}} = \frac{76}{2}$

$xy = 38$

x	y
2	19
-2	-19
1	38
-1	-38
19	2
-19	-2
38	1
-38	1



The polar coordinates of a point are given. Find the rectangular coordinates of this point.

$\;\;r\;\;\;\theta$
(7.1,2.7)

$x = r\cos\theta = 7.1\cos 2.7 = -6.4184$

$y = r\sin\theta = 7.1\sin 2.7 = 3.034$